

A discrete geometry model of fire propagation in urban areas

L-1 norms and fire propagation

Stéphane Gaubert and Daniel Jones

Stephane.Gaubert@inria.fr

daniel.jones@inria.fr

Inria, École Polytechnique

November 14, 2017

- ▶ $L = \mathbb{Z}^2$... the integer lattice
- ▶ A_k ... the set of points in $x \in L$ to which we can propagate in at most k time steps, $k \geq 1$ (and homogeneity of the system)
- ▶ $\tau_k(x) = \begin{cases} \text{propagation time to } x \text{ in at most } k \text{ steps, if } x \in A_k \\ +\infty, \text{ o.w.} \end{cases}$
- ▶ $v(x) = \tau^*(x) = \lim_{k \rightarrow \infty} \tau_k(x)$... propagation time to x
- ▶

$$f_k \equiv co(\tau_k) : dom(co(\tau_k)) \cup \mathbb{R}^2 \rightarrow \mathbb{R}$$

$$f_k \left(\sum_{i \in I} \lambda_i x_i \right) = \sum_{i \in I} \lambda_i \tau_k(x_i),$$

where $(\forall i \in I) x_i \in A_k$ and $\sum_{i \in I} \lambda_i x_i$ is a convex combination

Lemma

Let $x \in \text{dom}(f_k)$, then

$$f_k(x) = kf\left(\frac{x}{k}\right).$$

Lemma

The lower boundary of $k \times \text{epi}(f)$ is equal to f_k , where $k \times \text{epi}(f)$ denotes the k th Minkowski sum of $\text{epi}(f)$.

Lemma

$$k \times \text{epi}(x \rightarrow f(x)) = \text{epi}\left(x \rightarrow kf\left(\frac{x}{k}\right)\right)$$

Lemma

$$kf\left(\frac{x}{k}\right) \leq \tau_k(x) \leq kf\left(\frac{x}{k}\right) + \text{constant}.$$

It follows...

$$\inf_{k \geq 1} kf\left(\frac{x}{k}\right) \leq v \leq \inf_{k \geq 1} \left(kf\left(\frac{x}{k}\right) + \text{constant} \right)$$

Lemma

$$\inf_{k \geq 1} kf\left(\frac{x}{k}\right) = f'(0; x) = \sup_{p \in \partial f(0)} \langle p, x \rangle$$

Fire propagation is a polyhedral norm...

Theorem

$$\lim_{s \rightarrow \infty} \frac{v(sx)}{s} = \sup_{p \in \delta f(0)} \langle p, x \rangle .$$

The long-term geometry of the fire front depends simply on the immediate propagation directions, A_1 (since A_k are Minkowski sums of A_1)

Important example, the Von-Neumann neighbourhood...

$$A = \{(1, 0), (0, 1), (-1, 0), (0, -1)\},$$

with corresponding times $\tau_1, \tau_2, \tau_3, \tau_4$ respectively

- ▶ Radiative heating between large surface areas
- ▶ The polyhedral norm is a deformed L_1 ball

L_1 balls

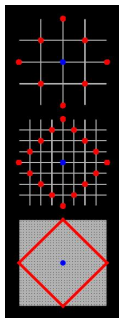


Figure: L_1 norm

- ▶ Q ... set of discrete states
- ▶ $Q = \{0, 1\}$ (ignited or not)... purely geometric
- ▶ For a simulation, we use the states used in the paper of Zhao

- ▶ 0 - original state (white)
- ▶ 1 - ignition
- ▶ 2 - flashover (self-developing)
- ▶ 3 - full development
- ▶ 4 - collapse
- ▶ 5 - extinguished

Show video 1

Non-perfect lattices and extra factors (wind or changing urban geometries)... deformed L_1 balls



Figure: L_1 balls

- ▶ Multiple sources... union of deformed L_1 balls (show video)
- ▶ Changing geometry across the urban environment... Finsler geometry

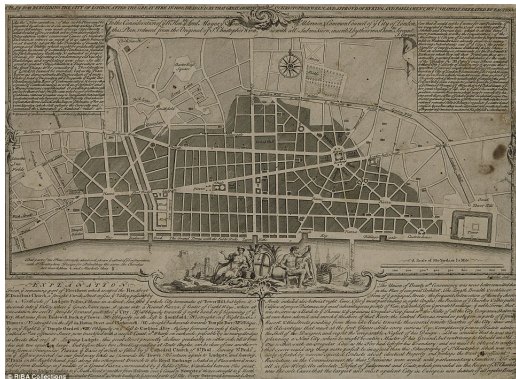


Figure: octagonal polyhedral norm

- ▶ change of wind on third day
- ▶ octagonal geometry on extreme edge of fire



Figure: 'dips' after crossing low density areas

- ▶ small 'dips' in the expected straight edges of the L1 ball
- ▶ modelled by a so-called 'Finsler-geometry'
- ▶ 'cell densities' can be incorporated into the model

future work

- ▶ 3D models for non-flat cities
- ▶ It may be possible to reverse engineer to find the ignition point
- ▶ Rome 64 AD, emperor Nero
- ▶ Different deformation of L_1 ball in different parts of the city
- ▶ Analytic formulas for propagation speeds to specific buildings
- ▶ Easier to interpret and modify than the applied model of Zhao
- ▶ Incorporate stochasticity (randomness) into the model (implies rounder corners of the fire front)

Thank you for your attention!

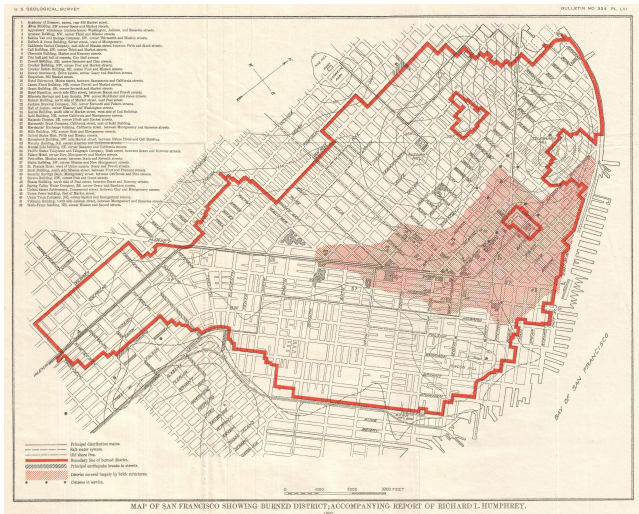


Figure: deformed L_1 ball with boundaries